Full counting statistics and dynamical phase transitions

The mandatory logos:

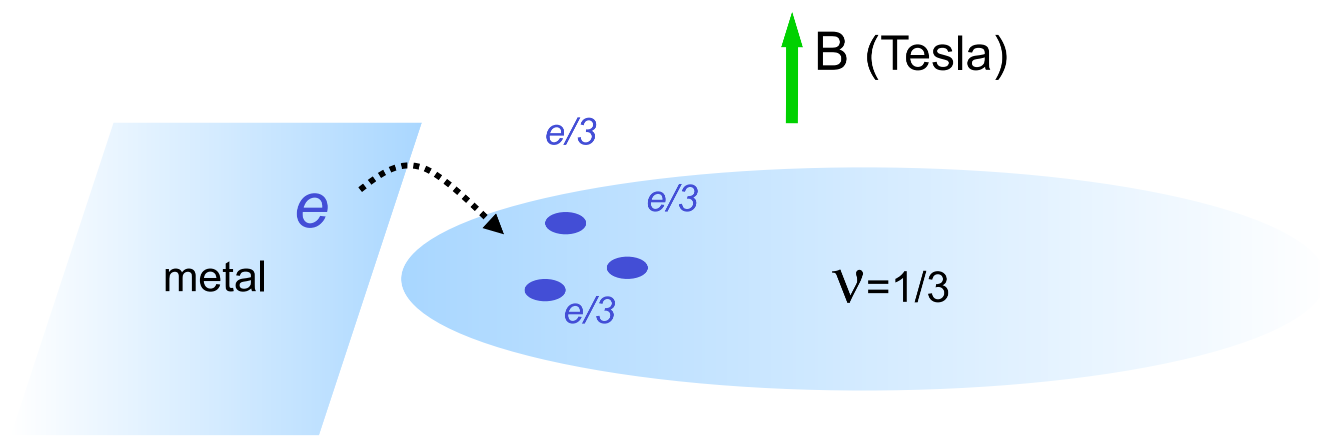
General question: features of the full counting statistics (ie, the probability distribution of charge transmitted) in quantum dots and similar devices in the presence of strong interactions, when a Fermi liquid picture does not hold.

work in progress

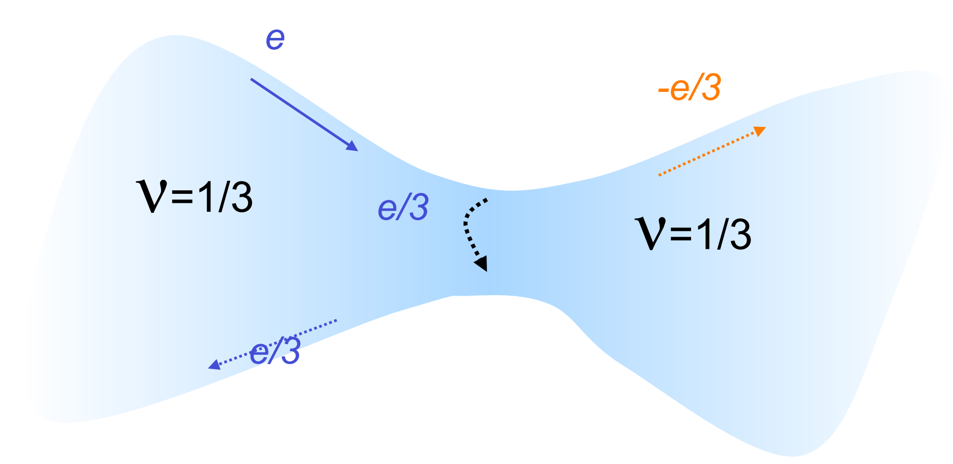
Probing LQP on the edge

- LQP which are gapped in the bulk are liberated at the edge. The gapless shape distorsions in the Hall fluid are excitations in a gas of fractionally charged QP

- Two possible set-ups

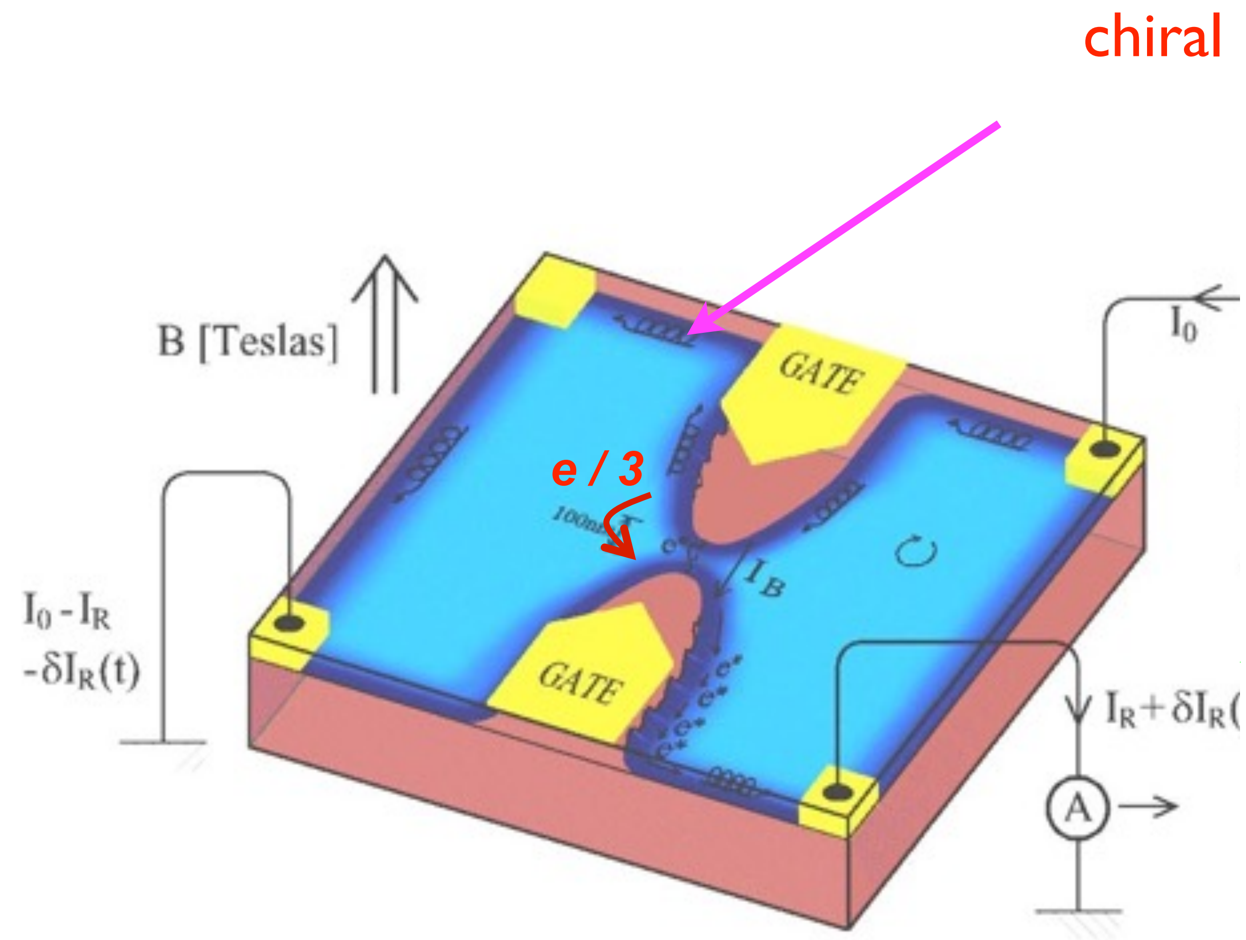


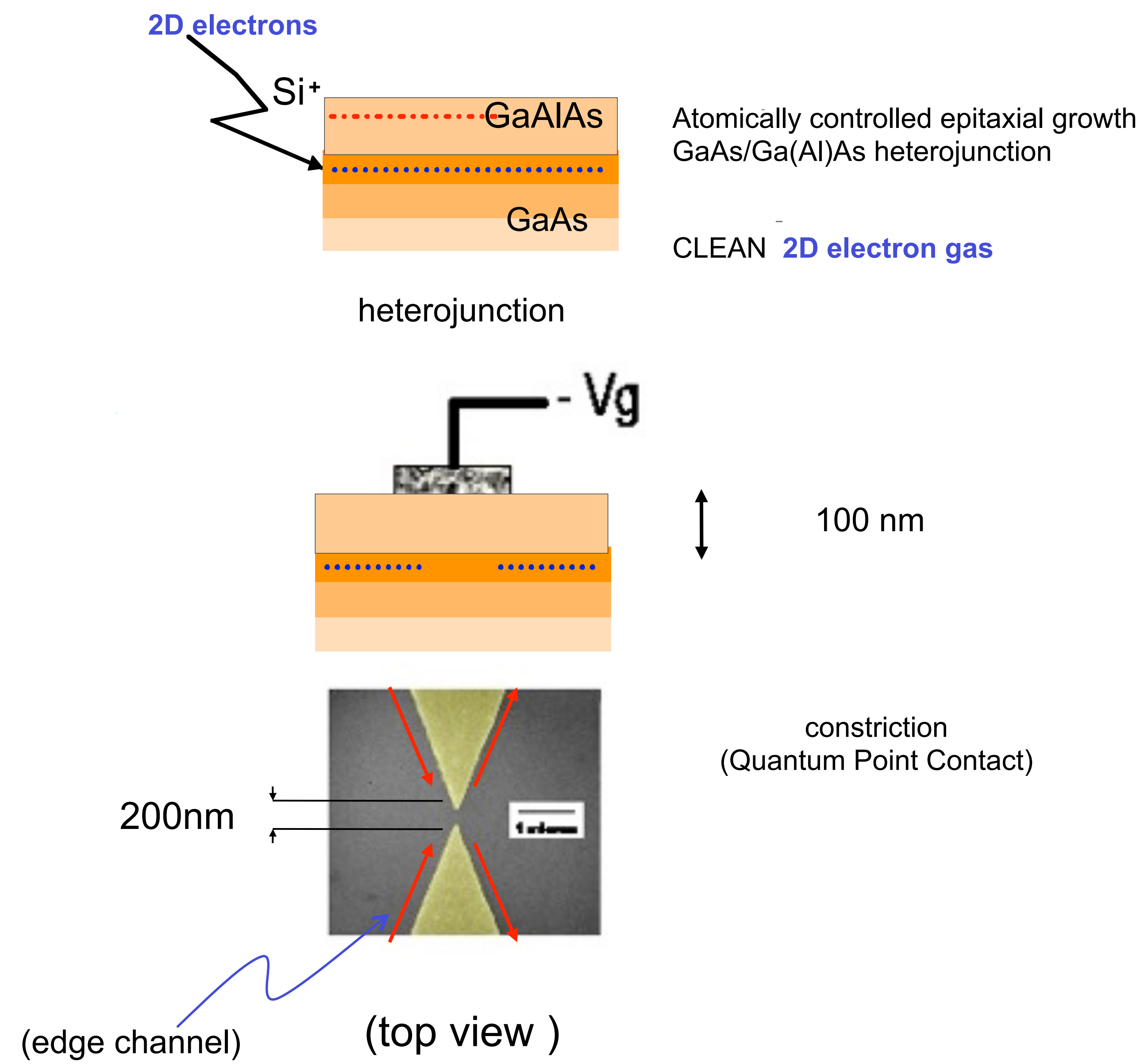
Chang et al. 1996



C. Glattli (Saclay) &
M. Heiblum (Weizmann)

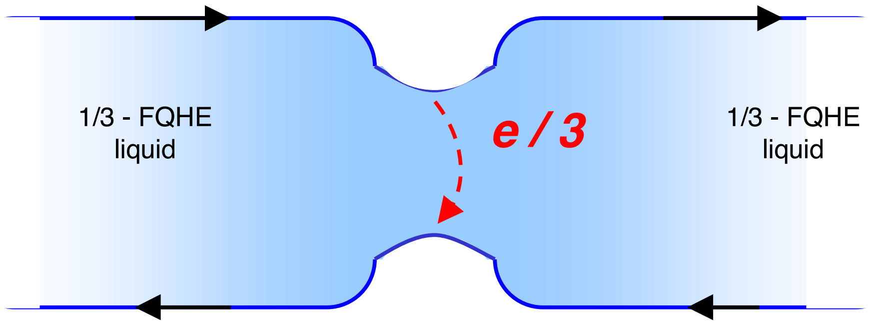
Tunneling between edges



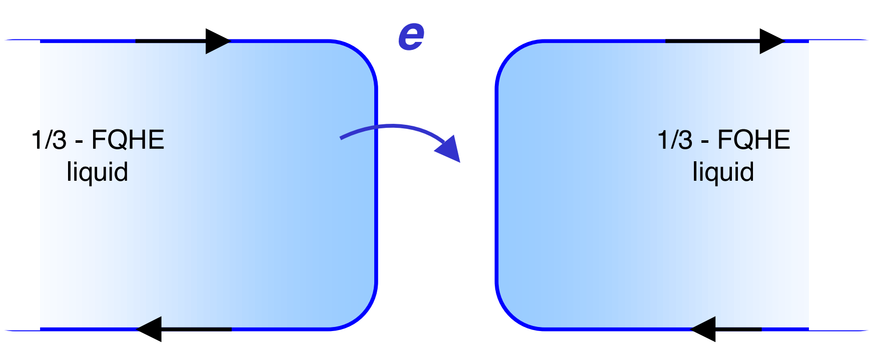


In a nutshell: Crossover

Weak back scattering



Strong back scattering



Kane & Fisher, 1992

Key idea: shot (Schottky) noise in the WBS limit should give access to charge of LQP

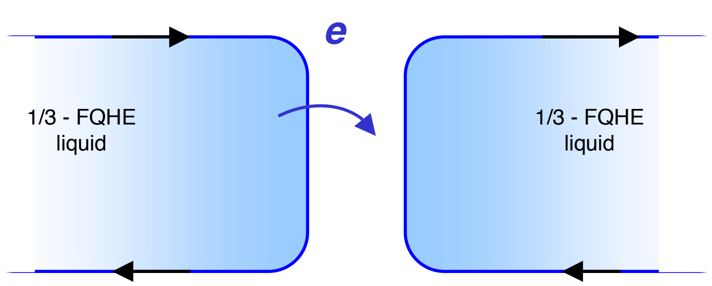
steady current measurements do not give access to the charge of the carriers

Generalizing Schottky formula

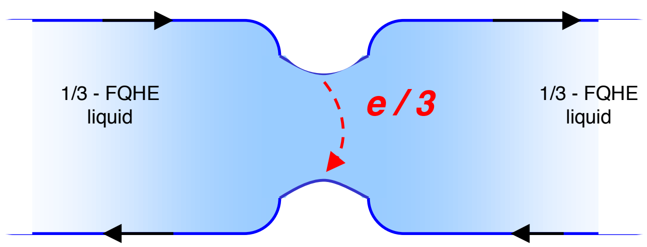
$$I << I_0$$

$$I_B = I_0 - I << I_0$$

SBS region



WBS region

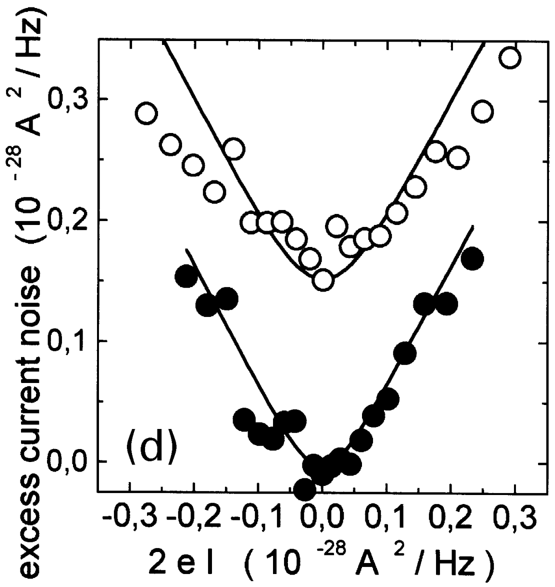


$$\langle (\Delta I)^2 \rangle = 2eI\Delta f$$

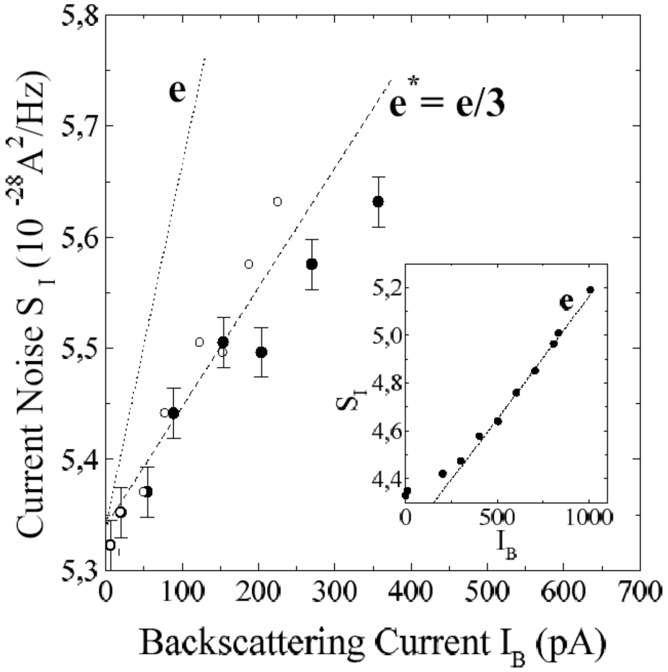
$$\langle (\Delta I)^2 \rangle = 2\frac{e}{3} I_B \Delta f$$

the lqp charge!!!

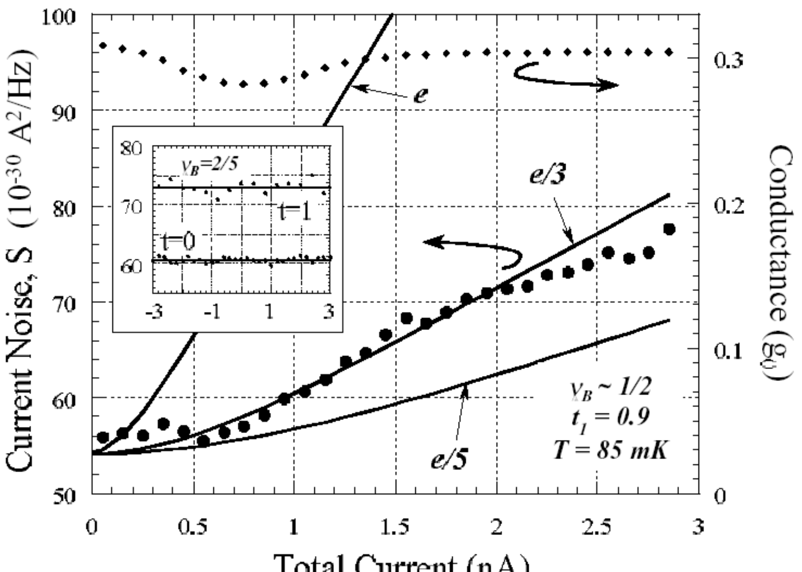
Beautiful experiments



D.C. Glattli, V. Rodriguez, H. Perrin, P. Roche, Y. Jin and B. Etienne, Physics E6 (2000) 22.



L. Saminadayar, D. C. Glattli, Y. Jin, and B. Etienne, Phys. Rev. Lett. 79, 2526 (1997).



R. de-Picciotto et al., Nature 389, 162 (1997).

Full counting statistics

- Generating function of cumulants of backscattered charge

$$Z(\chi) = \langle \Psi_0 | e^{i\chi \hat{Q}(t)} e^{-i\chi \hat{Q}(0)} | \Psi_0 \rangle \qquad \hat{Q}(t) = e^{i\hat{H}t}\hat{Q}(0)e^{-i\hat{H}t}$$

(Measure the charge backscattered at time 0 and again at time t   Levitov et al. 96)

- Naively, periodicity of the FCS in counting variable should change between WBS and SBS limits

does the FCS exhibit a phase transition in its analytic structure?

Ivanov & Abanov 10

The answer to this problem can be obtained (formally) in the scaling limit by solving the boundary sine-Gordon model out of equilibrium (Fendley Ludwig Saleur 95, Saleur Weiss 00, Bazhanov Lukyanov Zamolodchikov 98) at T=0

$$\frac{1}{t}\ln Z \to \frac{V}{2\pi}\sum_{n=1}^{\infty}\left(e^{i\chi n}-1\right)\frac{a_n(1/\nu)}{n}\left(\frac{V}{T_B'}\right)^{2n(\frac{1}{\nu}-1)}$$

SBS

!at large times!

$$\frac{1}{t}\ln Z \to \frac{V\nu}{2\pi}i\kappa + \frac{V\nu}{2\pi}\sum_{n=1}^{\infty}\left(e^{-i\chi\nu n}-1\right)\frac{a_n(\nu)}{n}\left(\frac{V}{T_B'}\right)^{2n(\nu-1)}$$

WBS

where

$$a_n(\nu) = (-1)^{n+1}\frac{\nu\sqrt{\pi}\Gamma(n\nu)}{2\Gamma(n)\Gamma(\frac{3}{2}+n(\nu-1))}$$

radius of convergence

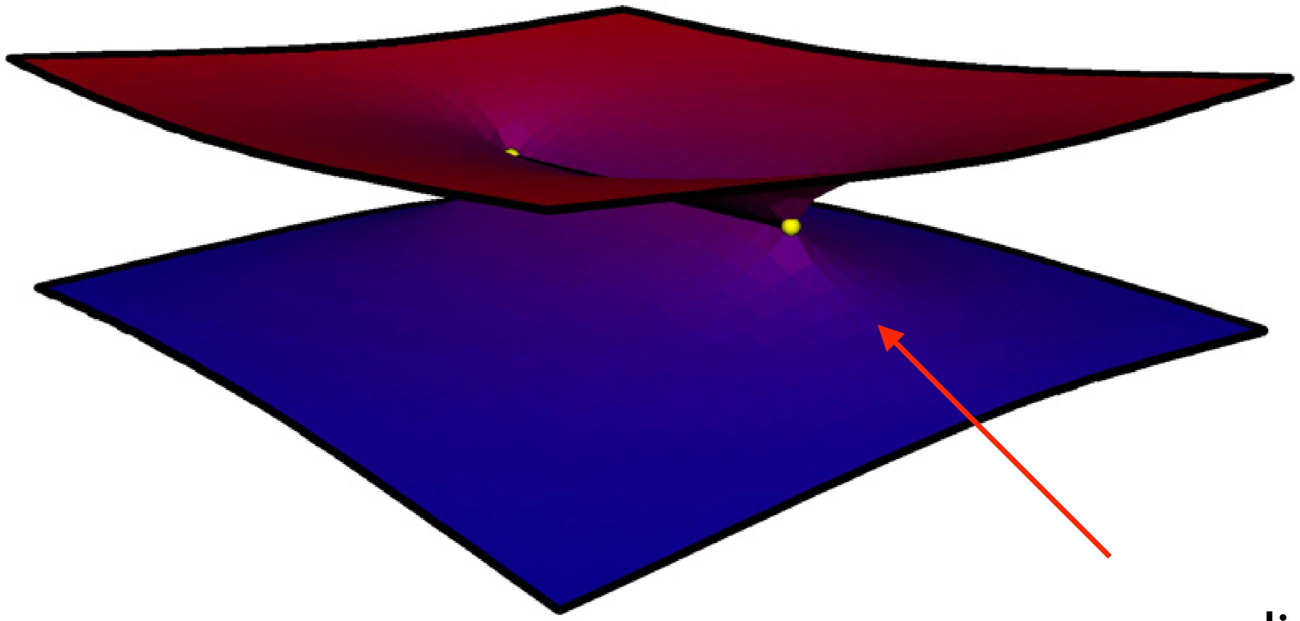
$V = V_c$

and T_B' is a crossover scale (like the Kondo temperature).

Observe the change of periodicity $e^{ikn} \to e^{-ik\nu n}$ for the counting current $I(\chi) = \frac{1}{t}\frac{\partial \ln Z}{\partial \chi}$

Of course nothing happens for the cumulants themselves!

- The analytical properties of this function are in fact well known (generalized hypergeometric functions (Fendley Saleur) . In the simplest case $\nu = 1/2$ (which is free), one has



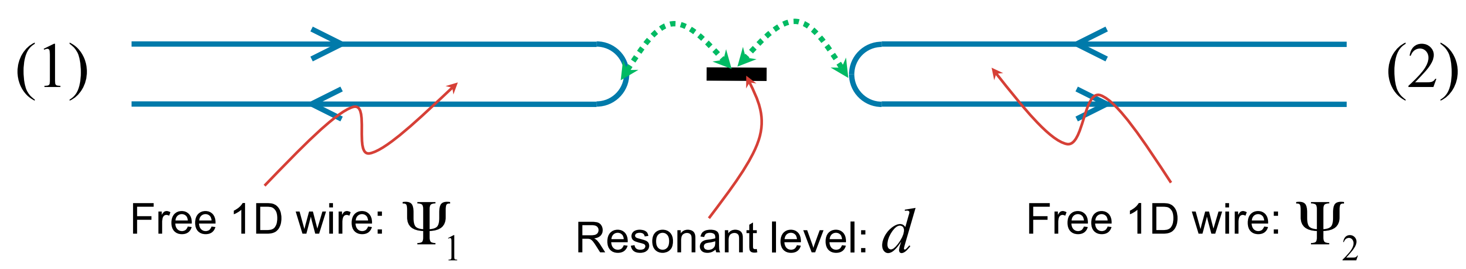
radius of convergence

$V = V_c$

What does ``really'' happen?

Some numerical studies

- For technical reasons it's been easier to study the IRLM



$$H_0 = -iv_F \sum_{a=1,2} \int_{-\infty}^{\infty} dx \ \ \Psi_a^\dagger \partial_x \Psi_a(x)$$

$$H_B = \left(\gamma_1 \Psi_1^\dagger(0) + \gamma_2 \Psi_2^\dagger(0)\right) d \ + \ U\left(:\Psi_1^\dagger\Psi_1:(0) + :\Psi_2^\dagger\Psi_2:(0)\right)\left(d^\dagger d - \tfrac{1}{2}\right) \ + \ \varepsilon_d d^\dagger d$$

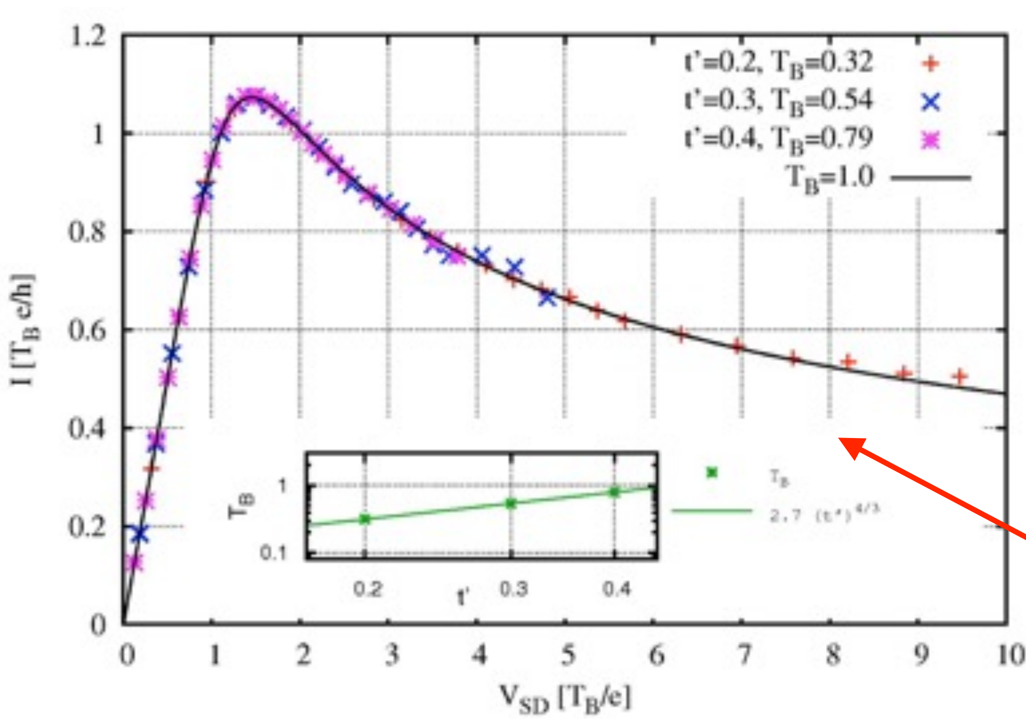
$\varepsilon_d=0$ at resonance

and its lattice discretizations. This is equivalent to BSG only for U=0 (free case) corresponding to $\nu = 1/2$ and $U = \pi$ corresponding to $\nu = 1/4$ formally (realized in fact in $\nu = 5/2$ case)

- Charge transmutation in this case is between e/2 tunneling at high-energy (large voltage or small tunneling amplitude) and 2e tunneling at low-energy.

The ratio $e^*_{HE}/e^*_{LE} = 1/4$ is equal to $\nu = 1/4$

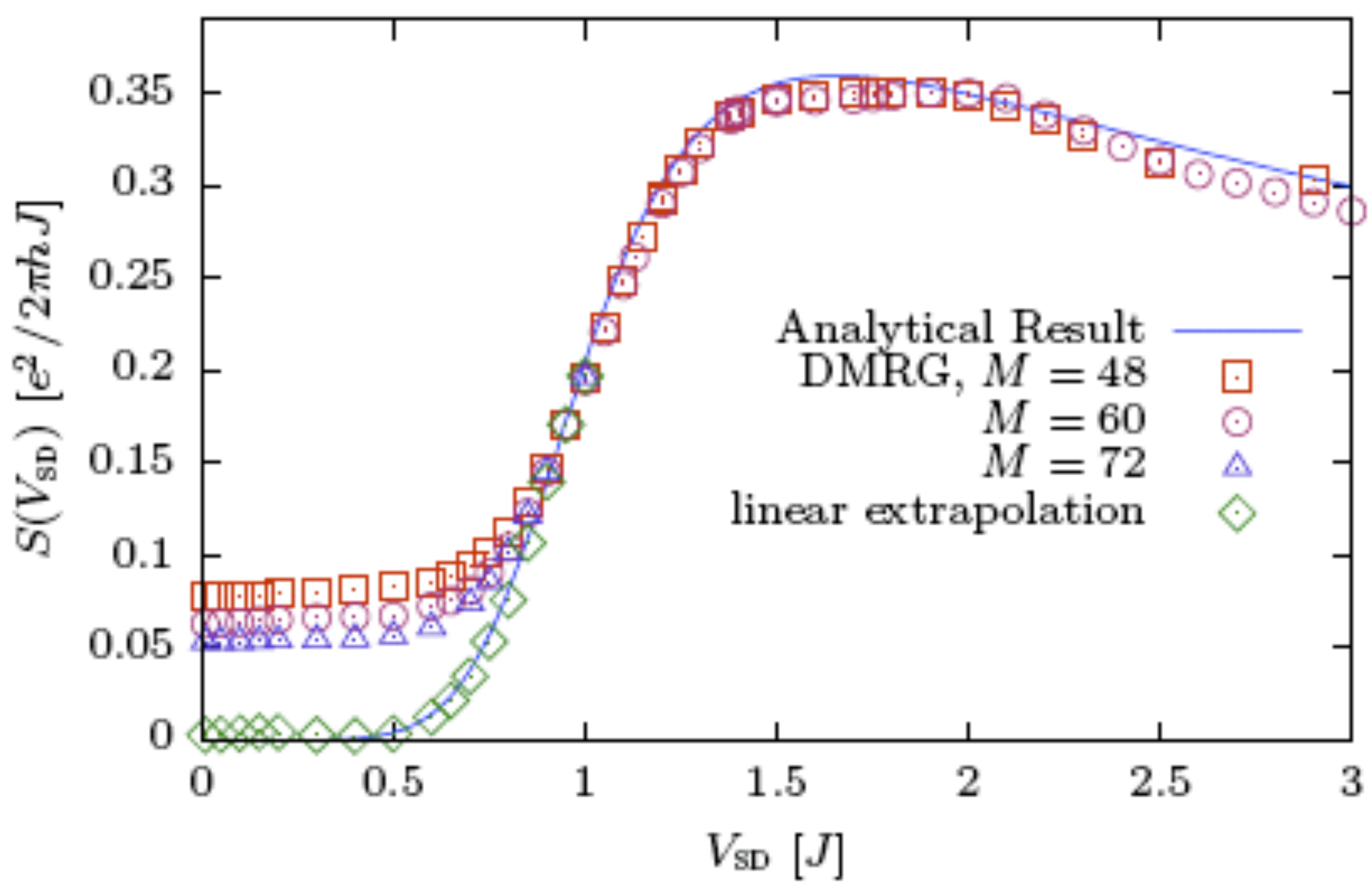
- A long collaboration with P. Schmitteckert in Karlsruhe has verified most of the analytical results.



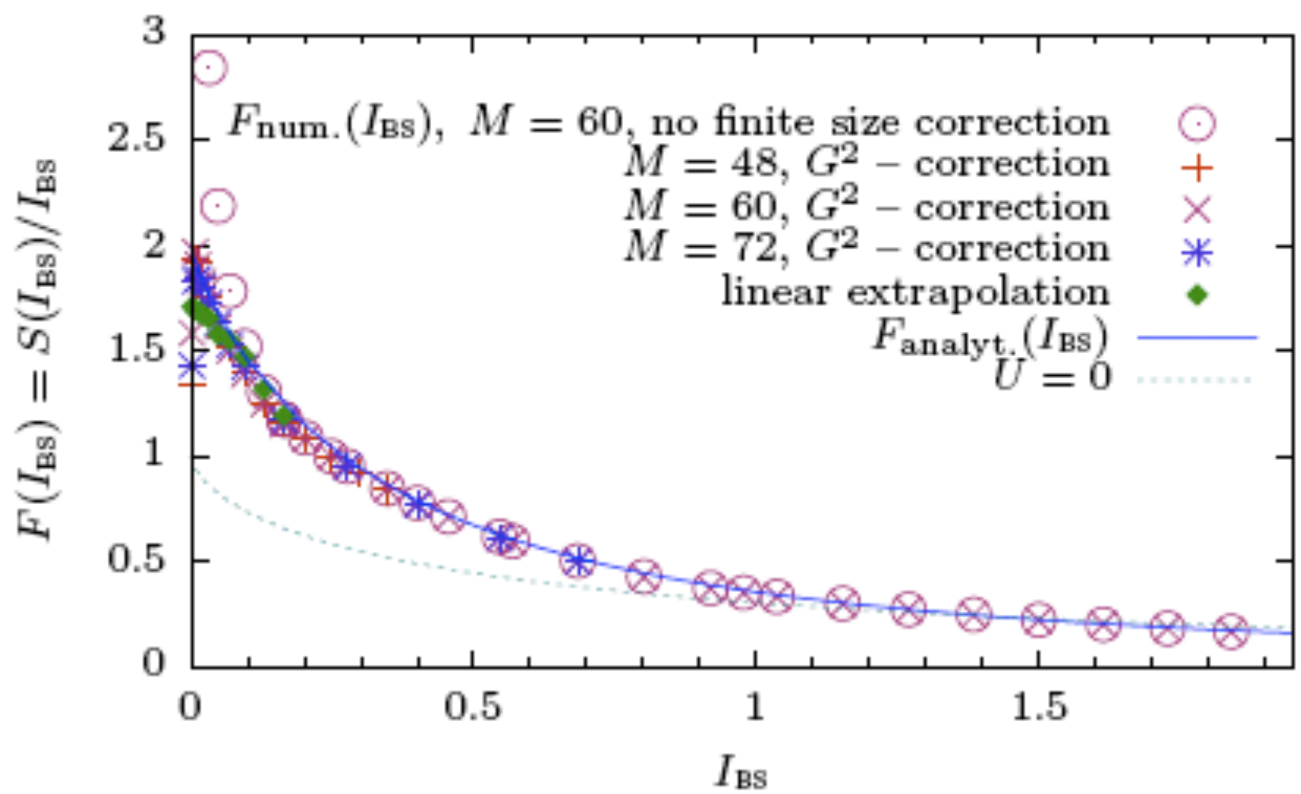
Things to check (among others): scaling as bare parameters become small +independence of details of conduction band + finite size effects

no singularity for the current

M=96 sites : N=2000 states kept



Boulat,Saleur,Schmitteckert



A technical remark: the formulas for the FCS now are

$$\frac{1}{t}\ln Z \to \frac{V\nu}{2\pi} i\kappa + \frac{V}{2\pi}\sum_{n=1}^{\infty}\left(e^{-i\chi n/(2\nu)} - 1\right)\frac{a_n(1/\nu)}{n}\left(\frac{V}{T'_B}\right)^{2n(\frac{1}{\nu}-1)}$$

$$\frac{1}{t}\ln Z \to \frac{V\nu}{2\pi}\sum_{n=1}^{\infty}\left(e^{-i\chi n/2} - 1\right)\frac{a_n(\nu)}{n}\left(\frac{V}{T'_B}\right)^{2n(\nu-1)}$$

Note that we have

$$\chi \to \frac{1}{2}\chi$$

(These expressions correspond to smooth continuation for small values of the counting parameter)

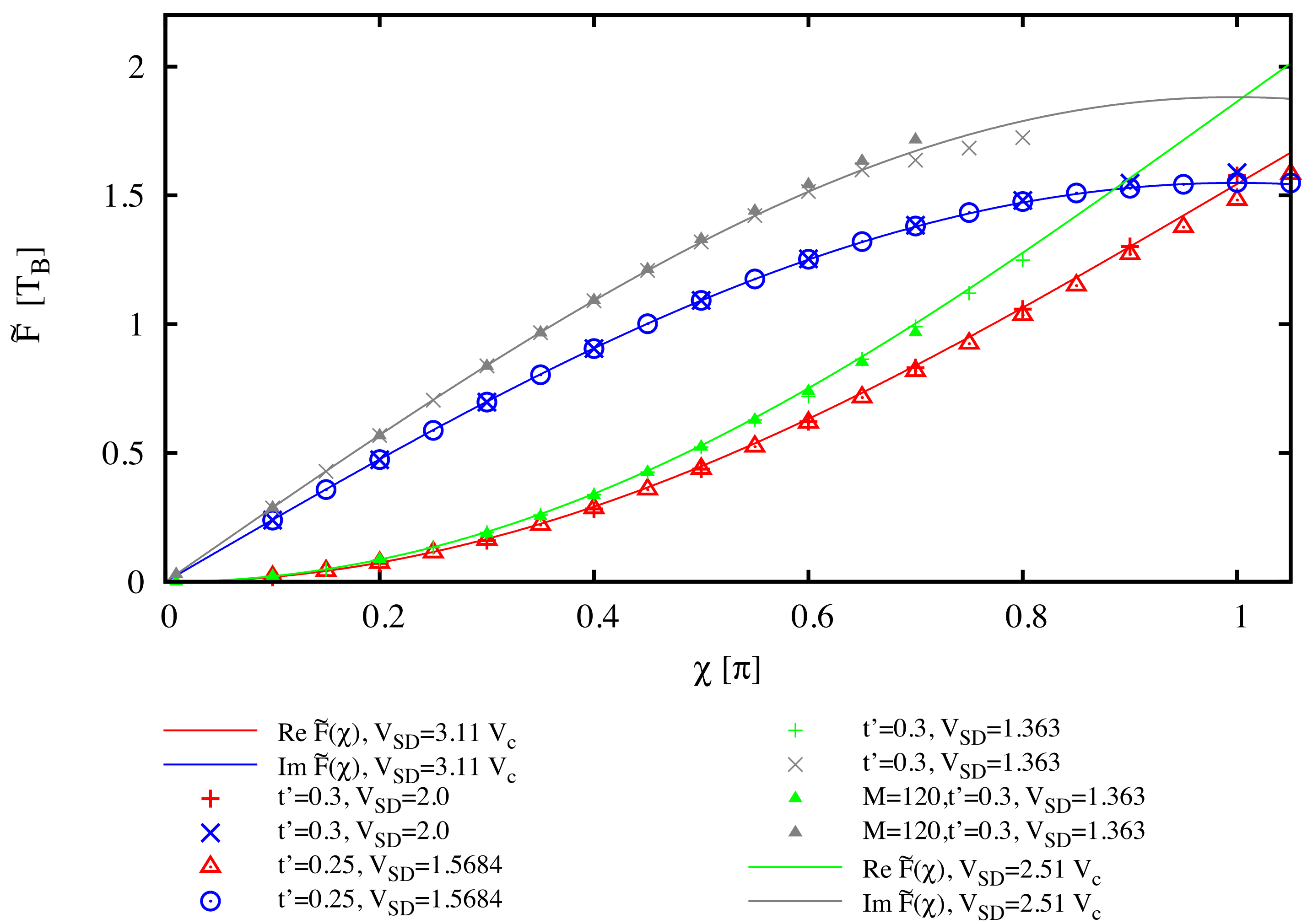
Period should go $4\pi \to \pi$ as one goes from weak to strong tunneling amplitudes.

More recently the FCS has become numerically accessible!!!

$$\mathcal{H} = -t \sum_{n=L,R} \sum_{i=0}^{L/2} \left(c^\dagger_{n,i} c_{n,i+1} + H.c. \right) + (\epsilon_0 - U) d^\dagger d$$
$$+ \sum_n \left(t'_n c^\dagger_{n,0} d + H.c \right) + U \sum_n \left(d^\dagger d - \frac{1}{2} \right) c^\dagger_{n,0} c_{n,0}$$

+ Bagrets, Carr

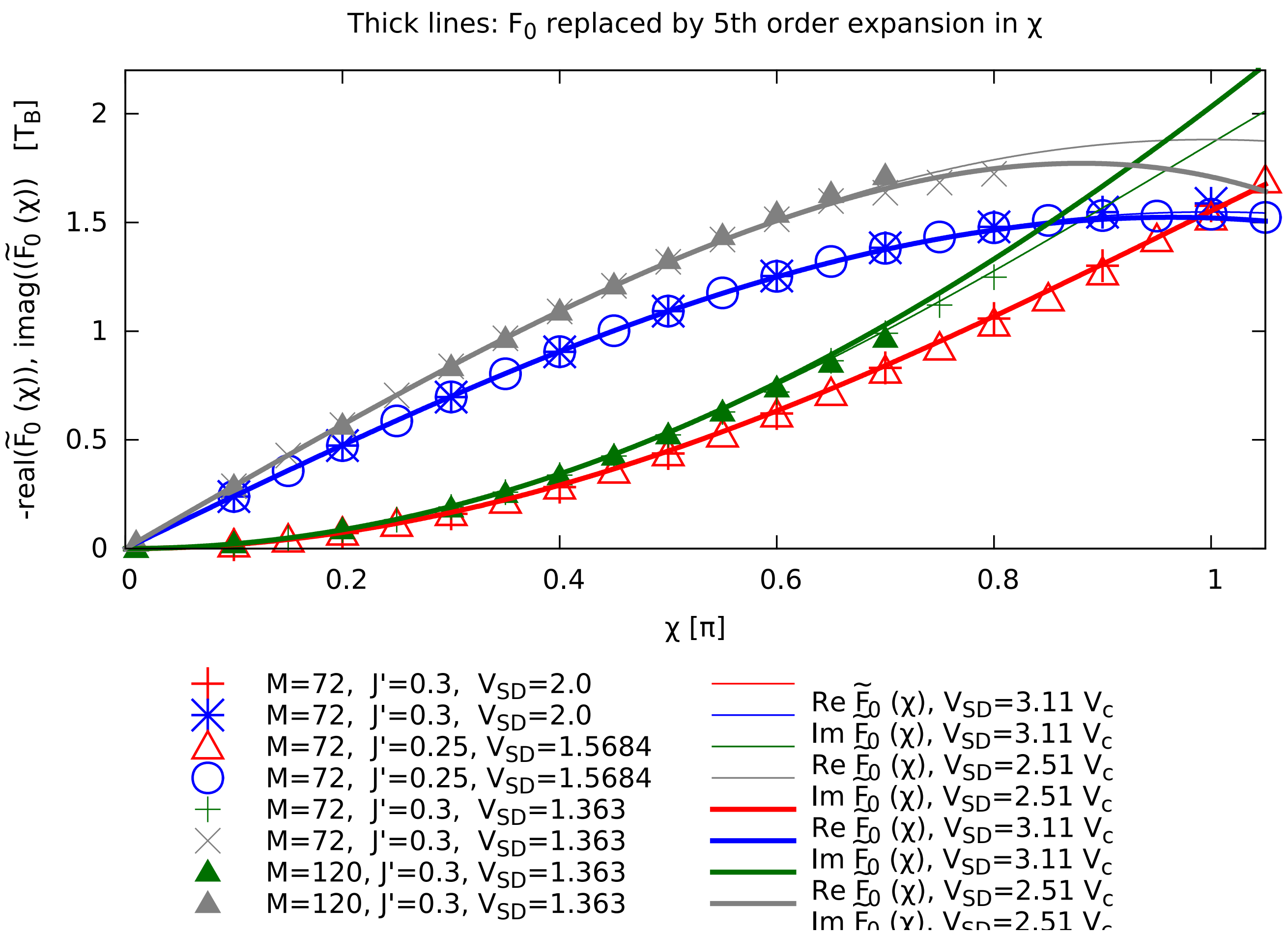
$t'_{L(R)} \rightarrow t' e^{\pm i\chi/4}$ $Z_{t_m}(\chi) = \langle \Psi(0) | e^{i\mathcal{H}_{-\chi} t_m} e^{-i\mathcal{H}_\chi t_m} | \Psi(0) \rangle$

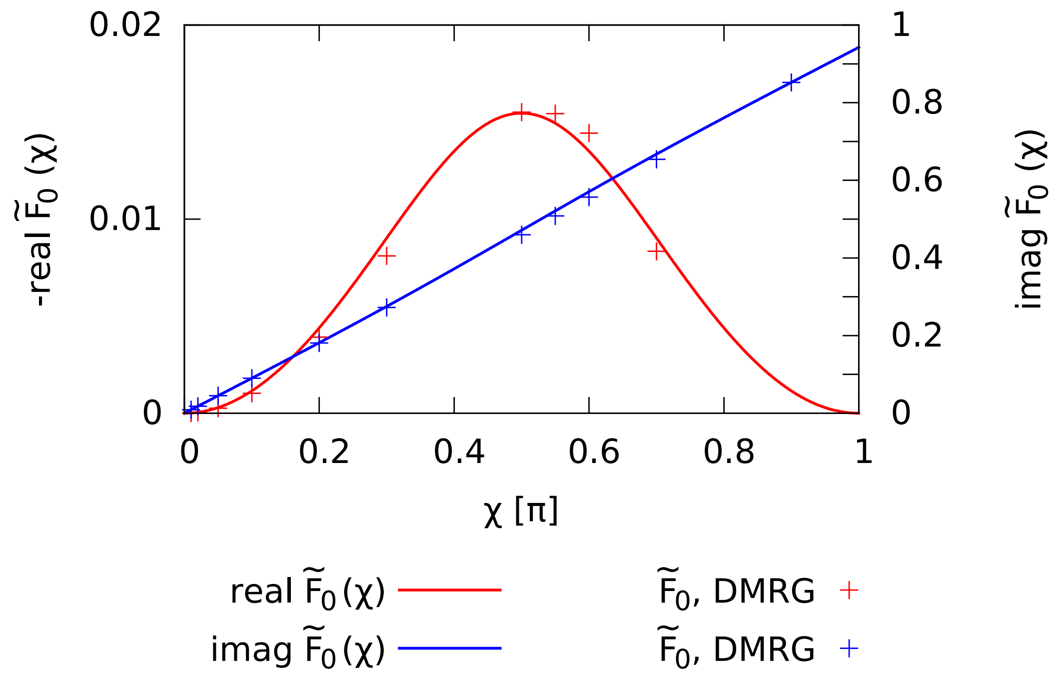


Once ν and T'_B are determined there's no fitting parameter at all.

Example in region $V > V_c$

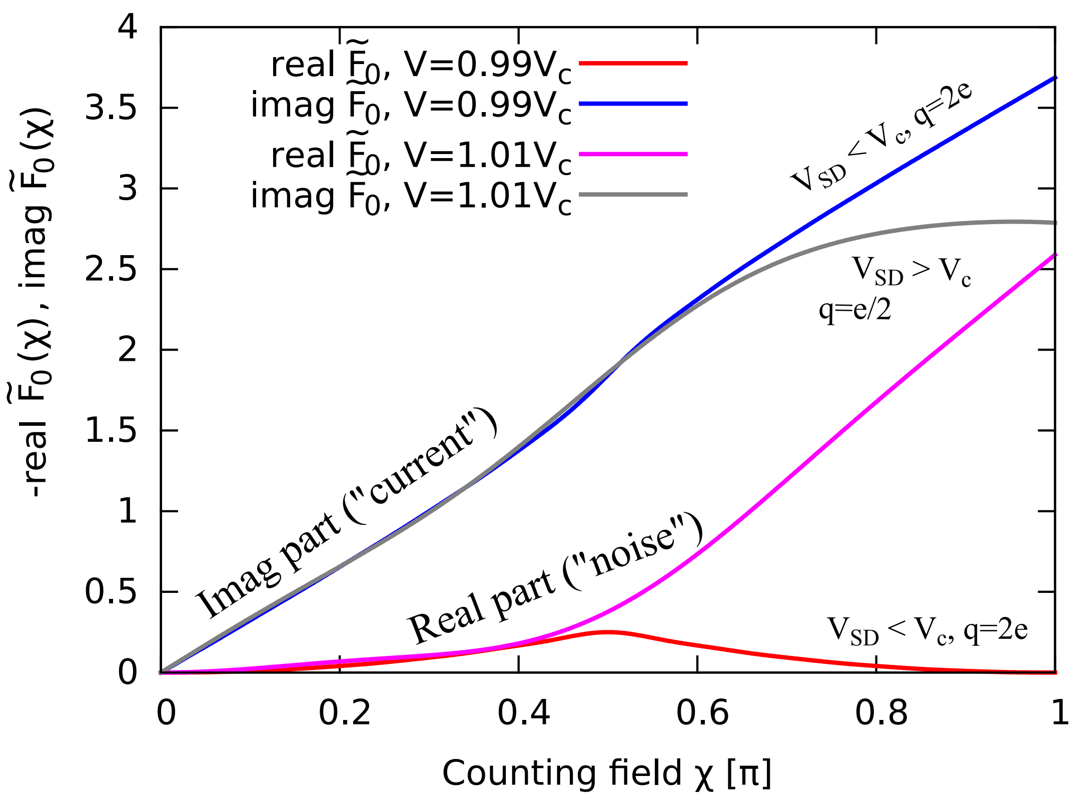
Note: one definitely sees more than current and noise!





Example in region $V < V_c \quad (V_c = 0.374, V = 0.3)$

- So we do see part of the change of (periodic) behavior in χ



Note that we have

$$\chi \to \frac{1}{2}\chi$$

in all numerical results
, so branch point is at $\frac{\pi}{2}$

Period

$$\pi \to 4\pi$$

Corrections to leading behavior

- It is however difficult to see the change of periodicity directly: data becomes extremely unstable beyond $\chi = \pi$. In fact, the question of long time corrections to the leading behavior plays a big ro
the study of a potential phase transition of the FCS

- We have strong numerical and analytical evidence that, at least for the BSG model

$$F = \ln Z(\chi, t) \approx \tilde{F}_0 t + \tilde{F}_1 \ln(Vt) + \dots$$

where moreover $\tilde{F}_1$ is universal.

Universal logarithmic corrections?

- Moreover we believe that

$$\tilde{F}_1 = \frac{2}{g} \left(\frac{d\tilde{F}_0}{dV} \right)^2$$

Where the formula comes from

Consider first a binomial process where a particle has probability p to tunnel. If n is the cha
transferred, we have

$$\langle e^{i\chi n}\rangle = 1 + p(e^{i\chi} - 1)$$

If we have N particles incident, and N also fluctuates,

$$\langle e^{i\chi Q}\rangle = \int p(N)dN \exp\left\{N \ln\left[1 + (e^{i\chi} - 1)p\right]\right\}$$

Now if p is Gaussian

$$p(N) \propto \exp\left[-\frac{(N - N_0)^2}{2\sigma^2}\right]$$

$$\langle e^{i\chi Q}\rangle = \exp\left\{N_0 \ln\left[1 + (e^{i\chi} - 1)p\right]\right\} \times \exp\left\{\frac{\sigma^2}{2}\ln^2\left[1 + (e^{i\chi} - 1)p\right]\right\}$$

In our problem, we will see that $N_0 \propto t, \quad \sigma^2 \propto \ln t$

U(1) charge fluctuations in one dimension $\propto \ln N$

For free fermions and energy independent scattering,

$$\langle e^{i\chi Q}\rangle \approx \exp\left\{\frac{tV}{2\pi}\ln\left[1+(e^{i\chi}-1)p\right]\right\}\times\exp\left\{\frac{\ln t}{4\pi^2}\times\ln^2\left[1+(e^{i\chi}-1)p\right]\right\}$$

(up to sub leading terms)

For free fermions and energy dependent scattering now ($\text{energy} \propto e^{\theta}$)

$$\langle e^{i\chi Q}\rangle \approx \exp\left\{tv_F\int_{-\infty}^{A}\rho(\theta)\frac{d\theta}{2\pi}\ln\left[1+(e^{i\lambda}-1)\tau(\theta)\right]\right\}\times\exp\left\{\frac{\ln t}{4\pi^2}\times\ln^2\left[1+(e^{i\chi}-1)p(A)\right]\right\}$$

Levitov Lesovik 93

the leading term is determined by low energy excitations, at the Fermi surface

Muzykantskii Adamov 03
Hassler Suslov Graf Lebedev Lesovik Blatter 08

In BSG, the semi classical description using integrable quasi particles has been successful - and got justified for the leading terms in the FCS. A naive extension would give the correction term

$$\exp\left\{\int_{-\infty}^{A}\int_{-\infty}^{A}\frac{d\theta d\theta'}{(2\pi)^2}C(\theta,\theta')\ln[1+(e^{i\chi}-1)\tau(\theta)]\times\ln[1+(e^{i\chi}-1)\tau(\theta')]\right\}$$

now complicated functions "determined" by Bethe ansatz

Using the known result

$$S(\omega)\propto\frac{1}{g}\left(\frac{dI}{dV}\right)^2$$

fixes the constraint

$$\int_{-\infty}^{A}\int_{-\infty}^{A}d\theta d\theta' C(\theta,\theta')\tau(\theta)\tau(\theta')=\frac{1}{g}\ln t\times\left(\frac{d}{dA}\int_{-\infty}^{A}\rho(\theta)\tau(\theta)d\theta\right)^2$$

a particular solution of which leads to

$$\exp\left\{\frac{1}{g}\ln t\times\left(\frac{d}{dA}\int_{-\infty}^{A}\rho(\theta)\frac{d\theta}{2\pi}\ln\left[1+(e^{i\lambda}-1)\tau(\theta)\right]\right)^2\right\}$$

that's the conjecture

Serious analytical checks

- Keldysh

$$Z(\chi) = 1 + \sin\frac{g\chi}{2}\sum_{m=1}^{\infty}(-1)^m\lambda^{2m}\int_0^t dt_{2m}\int_0^{t_{2m}} dt_{2m-1}\ldots\int_0^{t_2} dt_1$$

$$\sum_{\{\sigma_j\}}' \prod_{j=1}^{2m-1}\sin\left(\frac{g\chi}{2}+\pi g\eta_{j,2m}\right)\prod_{j>l}(t_j-t_l)^{2g\sigma_j\sigma_l}\prod_{j=1}^{2m}e^{-igV\sigma_j t_j}$$

$$\sum_{j=1}^{2m}\sigma_j = 0$$

the tunneling amplitude, $T_B' \propto \lambda^{1/1-g}$

$$\eta_{j,2m} = \sum_{k=j+1}^{2m}\sigma_k$$

Leading non trivial order involves four charges

only alternating charges contribute

$$\begin{cases} \mathcal{I}_1 \equiv \int (t_2-t_1)^{2g}(t_3-t_1)^{-2g}(t_4-t_1)^{-2g}(t_3-t_2)^{-2g}(t_4-t_2)^{-2g}(t_4-t_3)^{2g}\exp[-igV(t_1+t_2-t_3-t_4)] \\ \mathcal{I}_2 \equiv \int (t_2-t_1)^{-2g}(t_3-t_1)^{2g}(t_4-t_1)^{-2g}(t_3-t_2)^{-2g}(t_4-t_2)^{2g}(t_4-t_3)^{-2g}\exp[-igV(t_1-t_2+t_3-t_4)] \\ \mathcal{I}_3 \equiv \int (t_2-t_1)^{-2g}(t_3-t_1)^{-2g}(t_4-t_1)^{2g}(t_3-t_2)^{2g}(t_4-t_2)^{-2g}(t_4-t_3)^{-2g}\exp[-igV(t_1-t_2-t_3+t_4)] \end{cases}$$

and requires repeated use of stationary phase approximation

$$\int_\alpha^\beta e^{ixt}(t-\alpha)^{\lambda-1}(\beta-t)^{\mu-1}\phi(t)dt = B_N(x) - A_N(x) + O(x^{-N}), \quad x \to \infty$$

$$A_N(x) = \sum_{n=0}^{N-1} \frac{\Gamma(n+\lambda)}{n!} e^{i\pi(n+\lambda-2)/2} x^{-n-\lambda} e^{ix\alpha} \frac{d^n}{d\alpha^n}\left[(\beta-\alpha)^{\mu-1}\phi(\alpha)\right]$$

$$B_N(x) = \sum_{n=0}^{N-1} \frac{\Gamma(n+\mu)}{n!} e^{i\pi(n-\mu)/2} x^{-n-\mu} e^{ix\beta} \frac{d^n}{d\beta^n}\left[(\beta-\alpha)^{\lambda-1}\phi(\beta)\right]$$

before conjecture can be checked (it works).

- Case $g \to 0$: can be mapped onto a Langevin equation formalism

- Case $g = 1/2, \quad g = 1$ can be analyzed using a determinant formulation and the Fisher Hartwig conjecture

  Hassler Suslov Graf Lebedev Lesovik Blatter 08

  but there are subtleties near the transition point of the FCS.

- Details on the semi-classical case

$$\tilde{F}_0 = i\frac{Vg}{4\pi\sqrt{\pi}}\sum_{n=1}^{\infty}\frac{\Gamma(n-1/2)}{\Gamma(n+1)}\left(\frac{2\pi\lambda}{V}\right)^{2n}$$

SBS

$$\tilde{F}_0 = \frac{i\chi g}{2\pi}\left[V-\sqrt{V^2-(2\pi\lambda)^2}\right]$$

$$\tilde{F}_0 = i\frac{\chi g V}{2\pi}$$

WBS

Langevin equation

$$\dot{Q} = g\lambda\sin\left(gVt-2\pi Q+\xi(t)\right)$$

$$\langle\xi(t)\xi(t')\rangle = -g\ln|t-t'|$$

Fluctuations are Gaussian and affect only the second cumulant (the shot noise)

- Details on the non-interacting case

In fact the problem is well under control only for fluctuations in equilibrium

$$Z(\chi) = \langle e^{i\chi \sum_{i=1}^{L} c_i^\dagger c_i} \rangle = \det\left(\mathbf{1} + (e^{i\chi} - 1)\mathbf{g}\right) \qquad g_{i-j} = \frac{\sin(k_F(i-j))}{\pi(i-j)}$$

$\mathbf{T} \equiv \mathbf{1} + (e^{i\chi} - 1)\mathbf{g}$ is Toeplitz, with

$$T_{i-j} \equiv \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} e^{i\theta(i-j)} t(\theta) = \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} e^{i\theta(i-j)} \left[1 + (e^{i\chi} - 1)\Theta(k_F - |\theta|)\right]$$

Leading behavior is well known

$$\ln Z(\chi) \approx L \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} \ln t(\theta) = i\frac{k_F}{\pi}\chi L$$

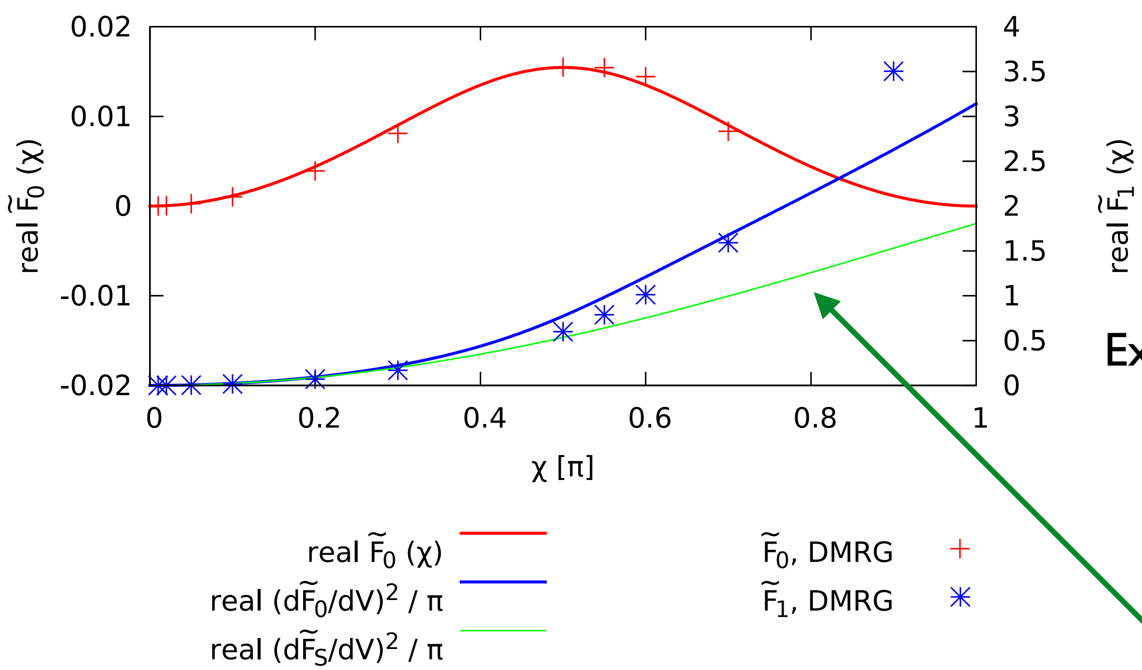
Abanov, Ivanov, Cheianov
Klich, Levitov, Lesovik

Corrections are described by Fisher Hartwig conjecture

Extension to transport case not so clear, at least for corrections (Hassler et al.)

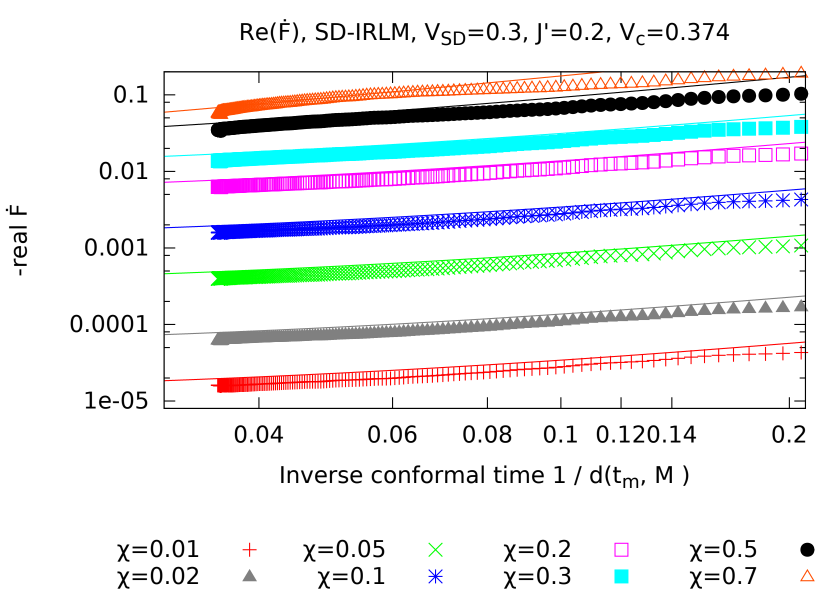
Numerical checks $g = 1/4$

Carr Saleur Schmitteckert 13,14



Example in region $V < V_c \quad (V_c = 0.374, V = 0.3)$

truncated to order χ^2

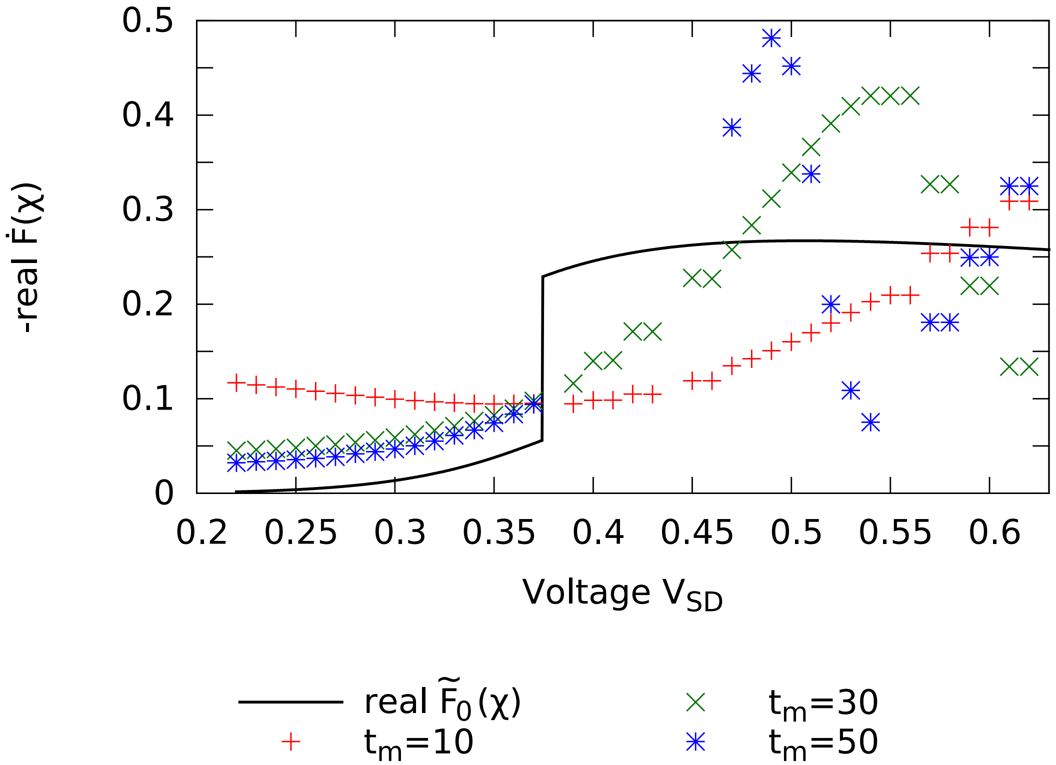


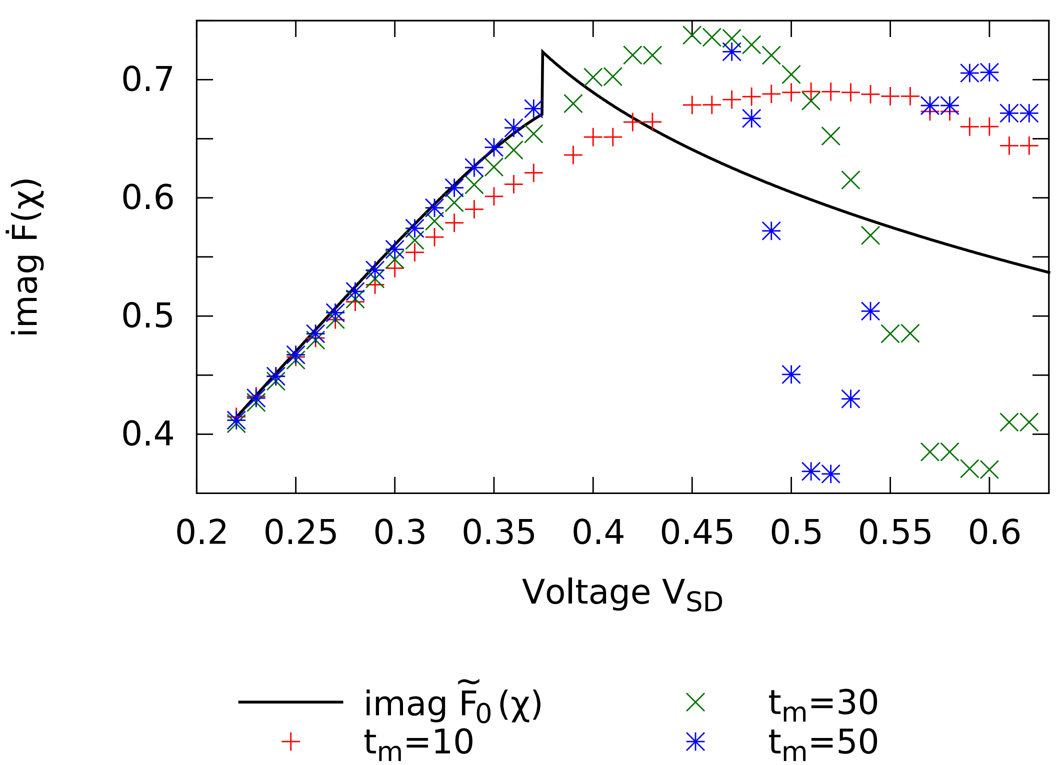
$$t_{\rm m} \to d(t_{\rm m}) = \left(\sin\frac{\pi t_{\rm m}}{M/v_c} - \sin\frac{\pi t_0}{M/v_c}\right)\frac{M\pi}{v_c}$$

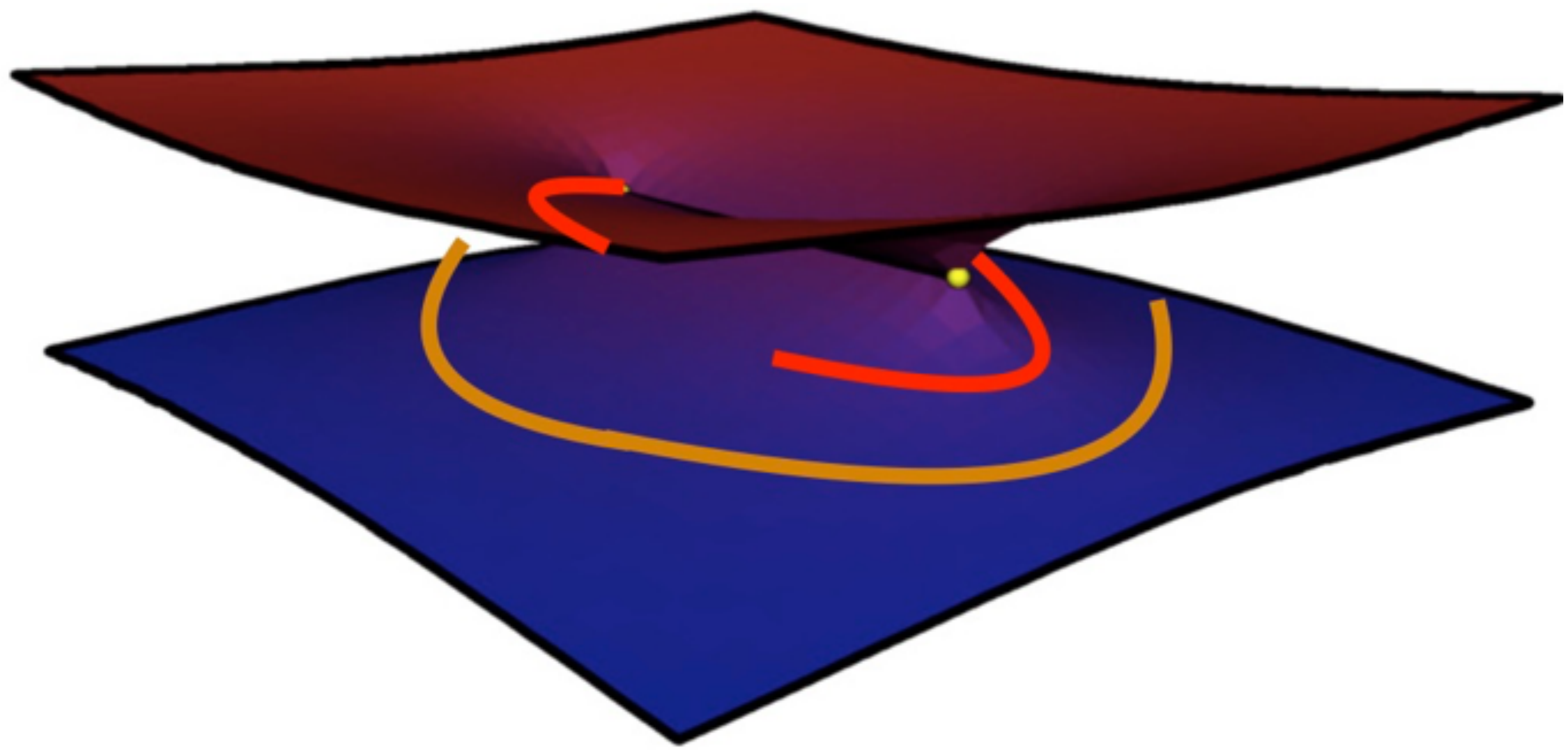
The formula however cannot work for χ large enough because of the singularity of the FCS

For a given χ, beyond V_c, we do see the change of sheet - albeit with strong oscillations

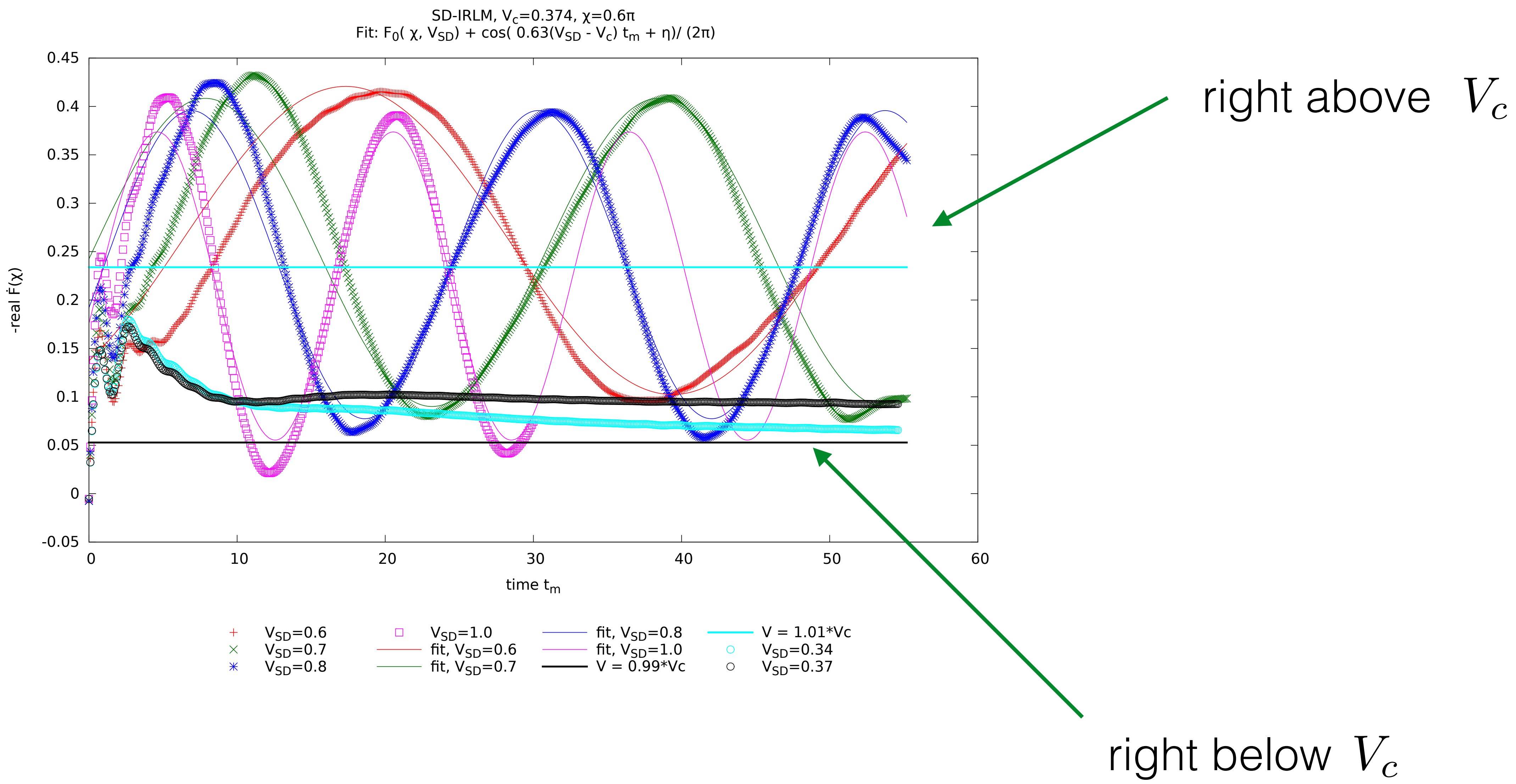
we do not understand







getting through **radius of convergence** $V = V_c$ for χ large enough



- The data is compatible with

$$\ln Z \sim \tilde{F}_0 t + \frac{A}{V - V_c} \cos\left[B(V - V_c)t + C\right]$$

so the FCS $\dot{F}$ is oscillating between the two main sheets

Conclusions

- The existence of universal $1/t$ corrections to the FCS in general seems reasonable

It is related with the logarithmic fluctuations of the charge in one dimension.

- The magical relationship with the FCS itself is probably only true in some integrable cases. We haven't even fully derived it however.

- Bifurcation of the FCS is a sign of transition between electrons and Lauglin quasiparticles. Not sure how it would be affected by non integrable terms. Case of finite temperature under investigation

- General question of Yang-Lee zeroes of FCS?